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Orbital elements for motion of real particle under the action of electromagnetic radiation

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Abstract. Discussion of different types of osculating orbital elements for motion of real dust particle under the action of electromagnetic radiation in the central gravitational field is presented. It is shown that physically correct access is based on gravitational acceleration as the only radial acceleration – "radiation pressure" is not included in the radial acceleration.

Key words: celestial mechanics, cosmic dust

1. Introduction

Orbital evolution of (interplanetary) dust particle under the action of central gravitational force and Poynting-Robertson effect (Robertson 1937, Klačka 1992a) was discussed in detail in Klačka (1992b – the statement "Equations (8) – (11) still hold." below Eq. (22) is incorrect, since Eq. (10) has to contain the product $\mu(1 - \beta)$ instead of μ). The paper was focused on two types of osculating orbital elements – the first type ("osculating elements I") was defined in the way that radial acceleration is given by central gravity force, the second type ("osculating elements II") was defined in the way that radial acceleration is given by central gravity force together with "radiation pressure" force. The advantage of the second type was discussed in Klačka (1992b, 1994). However, it

was stressed in Klačka (1992b), that osculating orbital elements of the second type "do not take into account correct physics". However, mathematics has significantly simplified for the second type of osculating orbital elements – secular changes of the "osculating" orbital elements can be easily calculated.

Klačka (2000) has derived equation of motion for general interaction between a particle and electromagnetic radiation. Consideration of gravitational field of the central body (Sun in the Solar System) leads, again, to the problem of defining of osculating orbital elements: they are defined in Klačka and Kocifaj (2001 – sections 3.2.1 and 3.2.2). Klačka and Kocifaj (2001) formulate both types of osculating orbital elements and they admit both of them as acceptable possibilities. However, some new results presented in the form of evolution of osculating orbital elements are not accepted by some astronomers dealing with interplanetary matter – they insist on using of the second type of osculating elements, i. e., the use of the osculating orbital elements II (section 3.2.2 in Klačka and Kocifaj 2001) is the only correct method according to their opinion.

The aim of this contribution is to look at the problem which type of osculating orbital elements is correct – if the "radial radiation pressure" has to be considered together with the central gravitational force or not.

2. Osculating orbital elements – definition

As a definition of osculating orbital elements we take the definition presented in Brouwer and Clemence (1961), since this book is very often used by scientists dealing with celestial mechanics and its application to Solar System. Brouwer and Clemence write (p. 273): "As the motion progresses under the influence of the various attracting bodies, the coordinates and velocity components at any instant may be used to obtain a set of six orbital elements. These are precisely the elements of the ellipse that the body would follow if from that particular instant on, the accelerations caused by all "perturbing" bodies ceased to exist." As for our purposes, it is sufficient to make a small change: "attracting bodies" are replaced by "forces". Our force coresponds to the electromagnetic radiation force.

3. Osculating orbital elements for particle interacting with electromagnetic radiation

Equation of motion for a particle in the central gravitational and electromagnetic radiation fields may be written (to the first order in $\boldsymbol{v}/c$ – higher orders are neglected, where

$\boldsymbol{v}$ is velocity of the particle, c is the speed of light) as

$$\frac{d\,\boldsymbol{v}}{d\,t} = -\,\frac{\mu}{r^3}\,\boldsymbol{r} + \frac{S\,A'}{m\,c}\,\left\{ Q'_R\,\left[\left(1 - \boldsymbol{v}\cdot\hat{\boldsymbol{S}}_i/c \right)\,\hat{\boldsymbol{S}}_i - \boldsymbol{v}/c \right] + \sum_{j=1}^{2} Q'_j\,\left[\left(1 - 2\,\boldsymbol{v}\cdot\hat{\boldsymbol{S}}_i/c + \boldsymbol{v}\cdot\hat{\boldsymbol{e}_j}/c \right)\,\hat{\boldsymbol{e}_j} - \boldsymbol{v}/c \right] \right\}\,, \tag{1}$$

where $\mu = G\,(M + m)$, G is gravitational constant, M is mass of the central body, m is mass of the particle, S is flux density of radiation energy, Q'_R, Q'_1, Q'_2 are "effective factors", A' is geometrical cross-section of a sphere of volume equal to the volume of the particle, $\boldsymbol{r}$ is position vector of the particle with respect to the body of mass M, $\hat{\boldsymbol{S}}_i \equiv \boldsymbol{r}/|\boldsymbol{r}|$ – for more details see Klačka (2000) or Klačka and Kocifaj (2001).

Eq. (1) may be rewritten in the form

$$\begin{aligned}
\frac{d\,\boldsymbol{v}}{d\,t} &= -\,\frac{\mu\,(1 - \beta)}{r^3}\,\boldsymbol{r} + \boldsymbol{a}_D\,, \\
\boldsymbol{a}_D &\equiv \frac{S\,A'}{m\,c}\,\left\{ -\,Q'_R\,\left[\left(\boldsymbol{v}\cdot\hat{\boldsymbol{S}}_i/c \right)\,\hat{\boldsymbol{S}}_i + \boldsymbol{v}/c \right] + \sum_{j=1}^{2} Q'_j\,\left[\left(1 - 2\,\boldsymbol{v}\cdot\hat{\boldsymbol{S}}_i/c + \boldsymbol{v}\cdot\hat{\boldsymbol{e}_j}/c \right)\,\hat{\boldsymbol{e}_j} - \boldsymbol{v}/c \right] \right\}\,, \\
\beta &\equiv Q'_R\,\frac{S\,A'}{m\,c}\,/\,\frac{\mu}{r^2}\,.
\end{aligned} \tag{2}$$

General opinion is that osculating orbital elements have to be taken with respect to the total central acceleration – given by the factor $\mu(1 - \beta)$. This corresponds to disturbing acceleration $\boldsymbol{a}_D$ in Eq. (2). However, this access is not correct. We want to show this. Let the disturbing acceleration $\boldsymbol{a}_D$ ceases at any instant. The problem is, that solution of Eq. (2) for $\boldsymbol{a}_D = 0$ does not correspond to ellipse (or to any kind of conic section) – $\beta-$ parameter is not a constant and its value changes during the motion even for the case $\boldsymbol{a}_D = 0$. As a consequence, the only correct form of defining osculating orbital elements is defined by central acceleration given by the factor μ.

4. Discussion

If we want to use osculating orbital elements in the way as they are defined in celestial mechanics, we have to use the central unperturbed acceleration given by gravitational acceleration of the central body – radial component of the radiation pressure cannot be included. Since the values of orbital elements exhibit a large dispersion during a revolution around the central body (Klačka 1994), it is wise – e. g., for the purpose of figures – to make time average (say, from pericenter to the following pericenter).

Of course, it is possible to use the central acceleration containing also radial radiation pressure. However, in this case, it is not sufficient to use only 'orbital' elements for

finding $\boldsymbol{r}$ and $\boldsymbol{v}$ – we need also to know β for a given instant; 'orbital' elements have no sense of osculating orbital elements defined in celestial mechanics. Time averaging is also recommended.

5. Conclusion

The problem of osculating orbital elements for the case of motion of a particle under the action of electromagnetic radiation was presented. We have shown that dealing with osculating orbital elements as defined in celestial mechanics, there is only one correct definition, in general case: central acceleration is given by gravitational acceleration, radial radiation pressure is not included.

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